A comparison of stock return distribution models

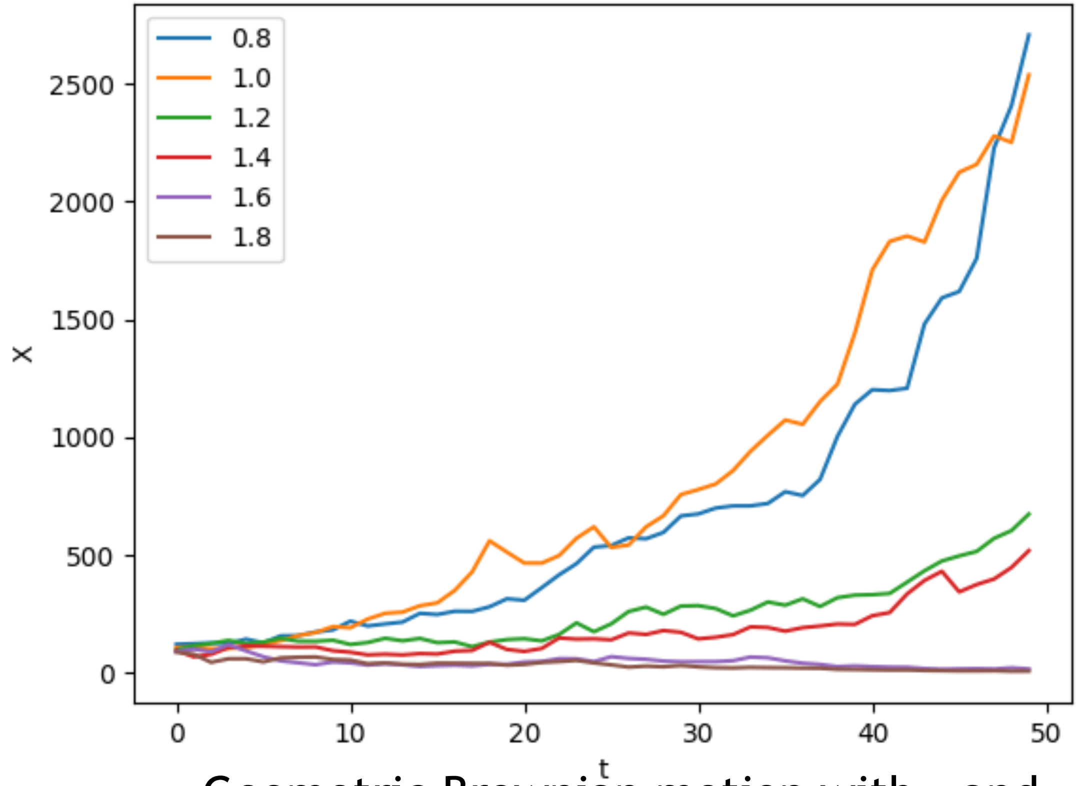
Jacqueline Garrahan

Modeling Stock Returns

- Significant body of research re market forecasting
- Stochastic models are used to capture random events
- Investment portfolios often developed around simulation outcomes
- I replicated two models used in prior work (QHO and GBM), and propose a third modified model (GBM ft. Ornstein-Uhlenbeck term)
- Applied to S&P 500 index (1-day returns & 20 day returns)

Geometric Brownian Motion

- Stochastic process (specifically Brownian motion) with drift and volatility parameters
- Used to model stock prices in the Black-Scholes model
- Widely used standard for modeling stocks

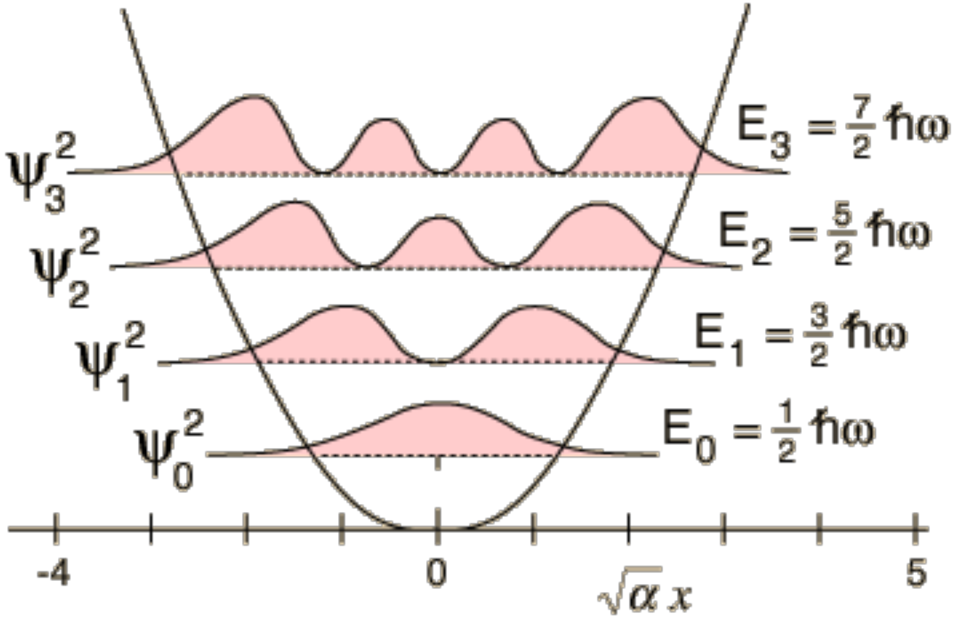


Geometric Brownian motion with , and different variances

https://en.wikipedia.org/wiki/Geometric_Brownian_motion

Quantum Harmonic Oscillator

- Ahn and colleagues propose the quantum harmonic oscillator for modeling stock returns
- Motivation is to capture local oscillations due to restorative market forces after big events
- The wavefunction of the quantum harmonic oscillator is a probability amplitude



Wavefunctions of the Quantum Harmonic Oscillator

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + \frac{1}{2}m\omega^2 x^2 \psi(x) = E\psi(x)$$

http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/hosc5.html

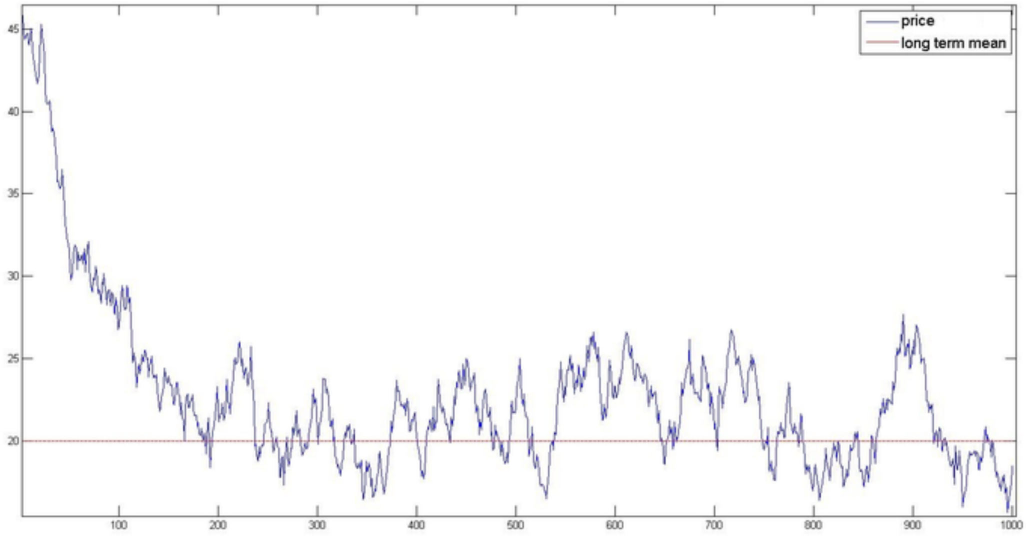
Model formulation

In order to capture the same oscillatory behavior, incorporated a term from a mean-reverting stochastic process.

Geometric Brownian Motion:

Ornstein-Uhlenbeck process:

Modified model:



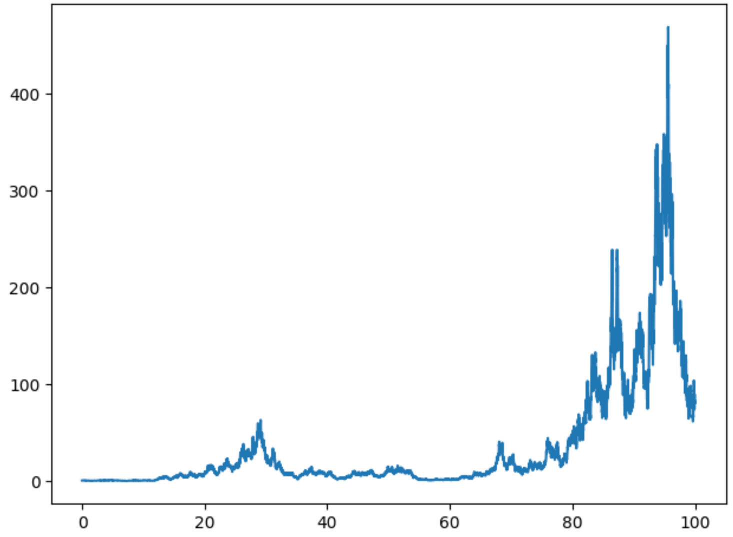
The Ornstein-Uhlenbeck process

https://www.researchgate.net/figure/Trajectory-Ornstein-Uhlenbeck-process-observe-the-reversion-rate-b-3-towards-the_fig1_281965169

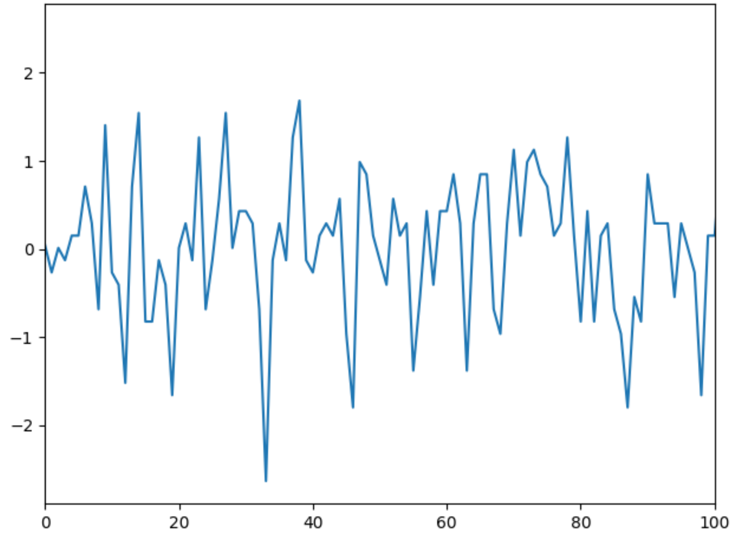
Methods

- Created a framework for simulating each of the models
- Searched against first five eigenstates of the QHO, variance and drift for the GBM, and the two variance parameters, mean inclusion window (how many recent vals to account for), and alpha
- Performed parameter search against S&P 500 index returns from last five years (particle swarm, simulated anneal). This searched against minimizing the Cramér von Mises test statistic.
- Compared test statistic for best parameter fit of each model.

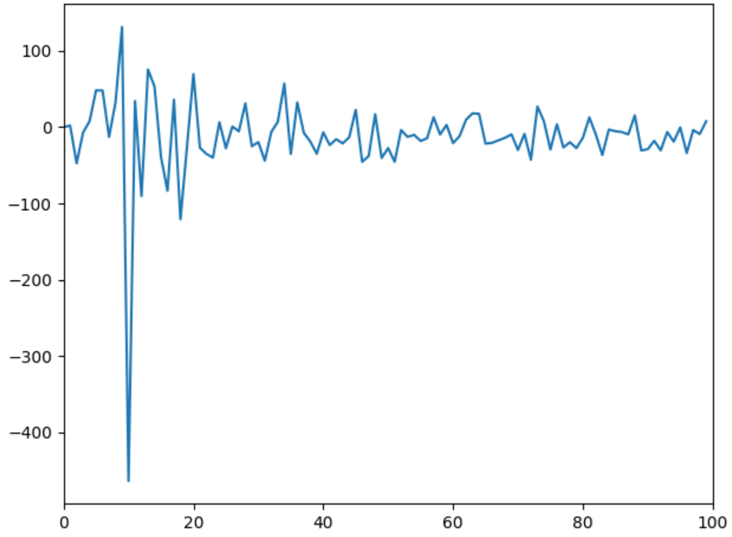
Results



Geometric Brownian Motion



Quantum Harmonic Oscillator



Modified Geometric Brownian Motion

Results

Models	Parameters	$\tau = 1$	$\tau = 20$
GBM	μ	-3.035	1.563
	σ	0.481	0.119
	T	239.404	151.625
Modified GBM	α	0	4
	σ_1	-0.2	-0.2
	σ_2	-0.2	-0.115
	μ_{steps}	20	50
	T	998.620	877.466
QHO	C_0	0	0.2
	C_1	0	0.2
	C_2	0.004	0.086
	C_3	0.053	0.182
	C_4	0.061	0.133
	$m\omega$	1	0.928
	T	296.607	552.782

Discussion/Conclusion

- Traditional GBM performs best; however, parameter search results are not super reliable due to computation demands of stochastic integration
  - Also some results had parameter-range edge cases, suggests better results with expansion
- The QHO also performs well for the 1-day returns
- Modified GBM accomplished mean reversion of the QHO
- Underscores options other than GBM, which is the standard tool used in deriving Black-Scholes etc.

Questions/Comments?